



Unit 2

SIMULATING THE TOSSES OF A FAIR COIN AND A FAIR DIE

Introduction

Simulation has become an important subject in industries, the medical field and other areas of human endeavour.

In this unit we use Microsoft Excel's random number function to simulate the tossing of coins, die and selection of items from containers.

Upon completion of this unit you will be able to:



Outcomes

- Write a Microsoft Excel program to simulate the tosses of a fair coin
- Write Microsoft Excel program to simulate the tosses of a fair die
- Relate relative frequencies to theoretical probabilities.



Terminology

simulation:

Using the computer to model a real life situation



Simulating the tosses of a coin

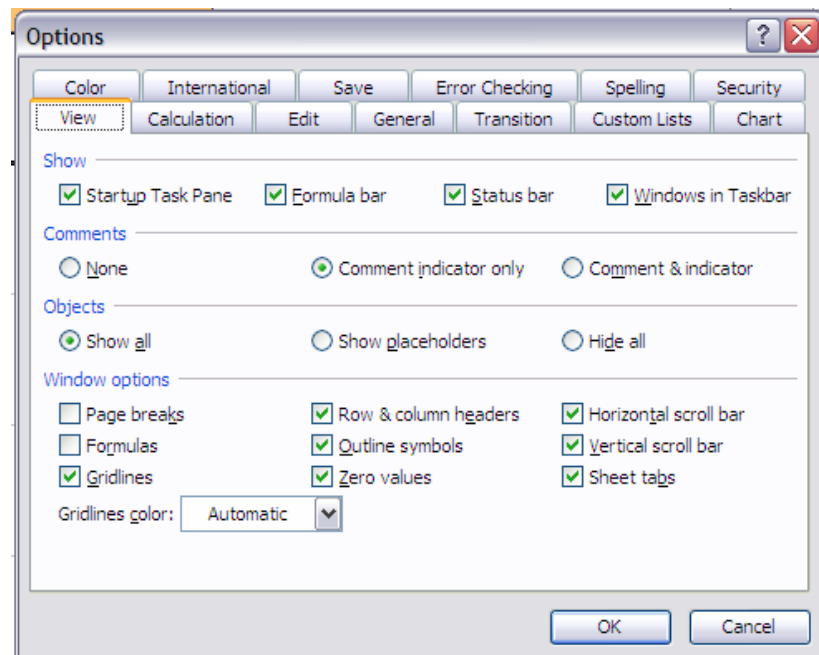
There are a number of inbuilt functions in Excel. One of them is the random number function. I suppose you have used random numbers before in selecting a sample from a population.

The random number formula =RAND() generates random numbers between 0 and 1

Let us assume that we are to toss a fair coin ten times (or ten fair coins once).

Again let us assume we want Excel to return 1 for heads and 0 for tails.

Open Excel and turn on Manual. To do this, click on **Tools** on the Menu bar. In the drop down list, click on **Options**. The dialogue below opens.



Click on Calculation then check the Manual radio box and click **OK** to return to the Excel worksheet.

We are turning on Manual so that Excel does not automatically calculate the formula we will enter when we copy into a new cell.

In order to solve the problem, let us create the headings below. Under Toss No. type 1 to 10 to indicate the ten tosses.

	A	B
1	Toss No.	Outcome
2	1	
3	2	
4	3	
5	4	
6	5	
7	6	
8	7	
9	8	
10	9	
11	10	

1. In cell B2 type =IF(RAND()<0.5,1,0)
(The formula =IF(RAND()<0.5,1,0) means that if the random number generated is less than 0.5, then Excel is to display 1 (meaning HEAD) in the cell otherwise it should display 0 (meaning TAIL)
2. Highlight cells B2 to B11 and choose Edit → Fill → Down
3. To run another simulation, press the F9 key on the key board



To run a series of simulations

1. Assume we want to run 10 of such simulations
2. Your heading will be like this:

	A	B	C	D	E	F	G	H	I	J	K
1	Toss No	Sim1	Sim2	Sim3	Sim4	Sim5	Sim6	Sim7	Sim8	Sim9	Sim10
2	1										
3	2										
4	3										
5	4										
6	5										
7	6										
8	7										
9	8										
10	9										
11	10										

3. In cell B2 type =IF(RAND()<0.5,1,0)
4. Highlight cells B2 to K11 and choose Edit → Fill → Down and Edit → Fill → Right

To calculate the number of heads and tails for each simulation we will use the FREQUENCY function

- a) Create the headings as shown below:

	A	B	C	D	E	F	G	H	I	J	K	L
15	Sim No	1	2	3	4	5	6	7	8	9	10	sum
16	0											
17	1											

- b) Highlight cells B16 and B17 and type
=FREQUENCY(B2:B11,\$A\$16:\$A\$17)
- c) Press simultaneously CTRL, SHIFT, ENTER
- d) Copy and paste in the rest of the cells (C16 to K17)
- e) In cell L16 type =SUM(B16:K16) and press the enter key

- f) In cell L17 type =SUM(B17:K17) and press the enter key
- g) Press the F9 key to get a set of simulated results.

Press the F9 key repeatedly. In each case observe the number of heads and the number of tails. Do you realize that the difference is not very much? This means the relative frequency of the occurrence of an event is very close to the theoretical probability. That is why probability is sometimes defined as **the expected long run relative frequency**.

Simulating the toss of a fair die

To simulate the toss of a fair die, we will use the formula
`=int(6*rand()+1`

We need to understand how the formula works

Let us start gradually and interpret the formula `int(6*rand()+1`. We know that the part in red will give us a number between 0 and 1. Multiplying by 6 gives us a number between 0 and 6. int then takes the integer part of the numbers between 0 and 6. That will be the numbers 0, 1, 2, 3, 4, 5. Finally we know that the number on the faces of a die are 1, 2, 3, 4, 5, 6 therefore adding 1 to 0, 1, 2, 3, 4, 5 will give us the required numbers.

Type the formula in a cell. Press the **Enter** key. To run a simulation press F9.



Unit summary



Summary

In this unit you learned how the random number function works. You also learnt how to use the random number function to simulate the tosses of a fair coin using the formula $\text{=IF}(\text{RAND()}<0.5,1,0)$. You also used the formula $\text{=int}(6*\text{RAND()}+1)$ to simulate the toss of a fair die.

Assignment



Assignment

1. Simulate the selection of a ball from an urn containing 7 white balls and three red balls. (Hint: Think of the white balls as numbered 1 to 7 and the red balls as numbered 8 to 10. Return the value W if a white ball is selected and R if red. The formula is $\text{=if}(\text{int}(10*\text{rand()}+1)\leq 7, \text{"W"}, \text{"R"})$)
2. Simulate the roll of a pair of fair dice ten times. (Hint: Use the formula $\text{=int}(6*\text{rand()}+1) + \text{int}(6*\text{rand()}+1)$)

Run 12 such simulations.

Draw a frequency table to illustrate the outcomes of the 12 simulations.

What are the probabilities of getting 2, 3, 4, ..., and 12?

How do they compare with the theoretical probabilities?

Draw a histogram for the distribution. How does it compare with the normal distribution?